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# Critical level statistics of a quantum Hall system with Dirichlet boundary conditions

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**Abstract.** We investigate numerically the influence of Dirichlet boundary conditions on the nearest neighbor level spacing distribution $P(s)$ of a two-dimensional disordered tight-binding model in the presence of a strong perpendicular magnetic field. From the calculation of the second moment of $P(s)$ it is shown that for Dirichlet boundary conditions, due to the presence of edge states, the position of the critical energy shifts with increasing system size to the location of the critical energy for periodic boundary conditions. An extrapolation to infinite system size results in different critical (scale independent) $P(s)$ distributions for periodic and Dirichlet boundary conditions.

**Keywords:** QHE, level statistics, critical spacing distribution, boundary conditions

## 1 Introduction

In recent years the statistics of energy eigenvalues has become an important tool for investigating the localization properties of disordered electronic systems [1, 2]. In most cases the Anderson model or one of its variants were used to describe the behavior of non-interacting electrons in three-dimensional disordered systems which possess orthogonal [3, 4, 5, 6, 7, 8, 9, 10], unitary [11, 12, 13], and symplectic [14] symmetry. Also two-dimensional systems with orthogonal [15, 16], symplectic [17, 18, 19], and unitary (random flux) [20] symmetry have been studied. Even in the quantum Hall case (strong magnetic field) [21, 22, 23, 24] level statistics was used to examine the localization properties.

It has been shown that in the limit of infinite system size, $L \to \infty$, the statistics of the uncorrelated eigenvalues in the insulating regime is specified by the Poisson form whereas in the diffusive regime the correlations of energy eigenvalues are well described [1] by random matrix theory (RMT), see e. g., [25, 26]. There are, however, also known deviations [2, 27, 28] from the universal RMT results. These deviations arise when the mean level spacing $\Delta$ is equal or larger than the Thouless energy $E_c = \hbar D/L^2$ which is the inverse of the classical diffusion time. The discrepancy is not really astonishing because the respective Hamilton matrix for the Anderson model with its large number of zeros is quite different from the random matrices considered in RMT [29, 30, 25].

Nevertheless, the success of extracting the critical properties that govern the localization-delocalization transition directly from level statistics [4, 6, 28, 24, 19] has stimulated detailed investigations of the new class of critical level statistics. In contrast to

the metallic and insulating phases where the level statistics assume their respective universal form in the limit of infinite system size, the critical P(s) was found to be scale invariant.

In particular, results on the energy spacing distribution of successive eigenvalues, $P(s)$, where $s = |E_{i+1} - E_i|/\Delta$, have been frequently reported in the literature. Until recently the form of the critical $P(s)$ was known only from numerical studies. However, the promising recent progress of analytical theories [31, 32, 33, 34, 35] will help to understand the critical eigenvalue correlations and to clarify the supposed relation [3] between the large-$s$ decay of $P(s)$ and the level number variance, $\Sigma^2(\langle N\rangle) = \chi\langle N\rangle$, which was shown to be connected with the multifractal exponent $D(2)$ of the critical eigenstates via $\chi = d - D(2)/(2d)$ [33].

Even in two-dimensional quantum Hall effect (QHE) systems, where the localization length diverges at singular energies in the center of the Landau bands and no complete localization-delocalization transition exists (absence of an energy range of extended states), critical level statistics can still be observed [21, 22, 23, 24].

Recently, the eigenvalue statistics directly at the transition was shown to depend on the boundary conditions in 3d orthogonal [36] and 2d symplectic systems [37], and also on the shape of the 3d sample [38]. For a QHE system, however, it has been suggested [39] that the dependence of the level statistics on the boundary conditions is absent when an appropriate shift of the critical energy is taken into account. In the following we address the question, whether the critical level statistics of the QHE system behaves differently to a change of the boundary conditions.

## 2 QHE-model and numerical method

The two-dimensional system of non-interacting electrons in the presence of impurity scattering and a strong perpendicular magnetic field can be described [40] by a tight-binding Hamiltonian on a square lattice with diagonal disorder. The magnetic field enters via complex phases in the transfer terms that cause the electronic motion within the $xy$-plane. In the Landau gauge the vector potential is taken as $A = (0, Bx, 0)$ which results in a magnetic flux density in the $z$-direction. The Hamilton matrix is

$$\begin{aligned}(H\psi)(x,y) = \epsilon(x,y)\,\psi(x,y) + V\,[\psi(x+a,y) + \psi(x-a,y) + \\ \exp(-i2\pi\alpha_B x/a)\,\psi(x,y+a) + \exp(i2\pi\alpha_B x/a)\,\psi(x,y-a)],\end{aligned} \tag{1}$$

where $a = 1$ and $V = 1$ are taken as the unit of length and energy, respectively. The magnetic field $B$ is chosen to be commensurate with the lattice and $\alpha_B = a^2 eB/h$ denotes the number of flux quanta per plaquette. The boundary conditions are changed from periodic (PBC) to Dirichlet (DBC) by setting $V = 0$ along the edges. The diagonal disorder potentials $\epsilon(x,y)$ are a set of independent random numbers drawn from an interval, $-W/2 \le \epsilon \le W/2$, with constant probability density $1/W$.

Square systems of size $L/a = 64$, 96, 128, 256 are considered with disorder strength $W/V = 2.5$. Taking $\alpha_B = 1/8$, the ratio of system size $L$ to the magnetic length $l = (\hbar/(eB))^{1/2}$ is about 201 for the largest samples. The corresponding large sparse hermitian matrices were directly diagonalized by means of a Lanczos algorithm based on [41] which was adapted to run efficiently on DEC-Alpha workstations. Many realizations of the disorder potentials were calculated so that the number of accumulated eigenvalues within a narrow interval around the critical energy of the lowest Landau band exceeded $2\cdot 10^5$ even for $L/a = 256$. A spectral unfolding procedure was applied in order to eliminate global changes in the density of states.

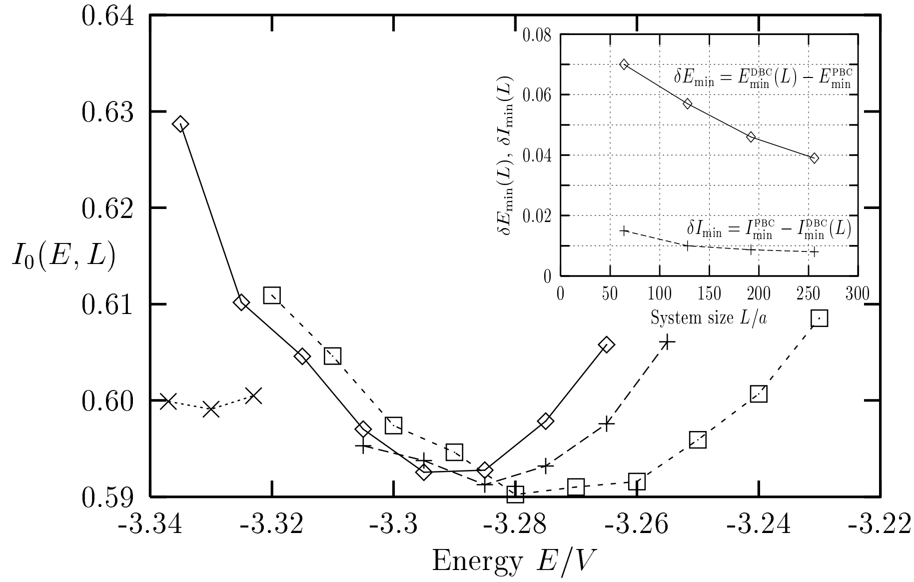


**Fig. 1:** The energy and size dependence of $I_0$ for DBC: ($\square$) $L/a = 128$, ($+$) $L/a = 192$, ($\diamond$) $L/a = 256$. In contrast, for PBC the energetic position of the $I_0$-minimum is size independent: ($\times$) $L/a = 128$. The lines only connect points that belong together. The inset shows the size dependent shift of the energetic position and the value of the $I_0$-minima. The minimum value of $I_0$ seems to saturate at about 0.592.

## 3 Results and discussion

As mentioned in the introduction, the critical disorder $W_c$ which defines the position of the divergence of the correlation length, $\xi(W) \sim |W - W_c|^{-\nu}$, does not change significantly in 3d orthogonal [36] and 2d symplectic [37] systems when the periodic boundary conditions (PBC) are replaced by Dirichlet boundary conditions (DBC). At $W_c$ and for finite systems, there exists a broad energy interval for which the localization length exceeds the system size $L$ so that these states show critical behavior. For the 2d QHE-System, on the other hand, the divergence of the localization length, $\lambda \sim |E - E_n|^{-\nu}$, takes place at the positions of the critical energies, $E_n(W)$, which depend on the disorder strength. Therefore, only a small critical energy window around $E_n$ exists where $\lambda(E) >> L$.

To determine the position of the critical energy in the lowest Landau band, $E_0$, we have calculated the energy dependence of the second moment $I_0(E,L)$ of the level spacing distribution, $I_0(E,L) = 1/2 \int_0^\infty s^2 P(s)\, ds$, for PBC and DBC. The minimum of $I_0$ indicates the position of $E_0$ which is shown in Fig. 1 for disorder $W/V = 2.5$. While the energetic position of the minimum of $I_0$ does not change with the system size $L$ for PBC, a pronounced shift is visible in the case of DBC. The reason for the shift originates in the edge states which are introduced by the application of DBC. However, their influence on the level statistics vanishes with increasing system size. This is shown in the inset of Fig. 1 where the difference $\delta E_{\min} = E_{\min}^{\rm DBC}(L) - E_{\min}^{\rm PBC}$ between the energetic positions of the minimum of $I_0$ for DBC and PBC is plotted. There, also the difference $\delta I_{\min} = I_{\min}^{\rm PBC} - I_{\min}^{\rm DBC}(L)$ in the corresponding values of $I_0$

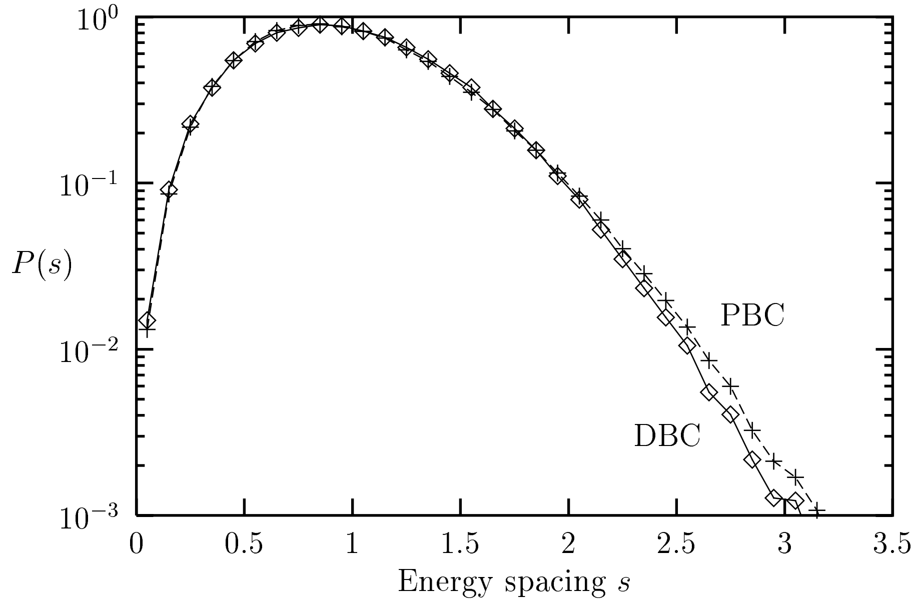


**Fig. 2:** The critical (scale independent) energy spacing distributions $P(s)$ for periodic boundary conditions (PBC, $+$) and Dirichlet boundary conditions (DBC, $\diamond$) versus energy spacing $s$ in the lowest Landau band.

is shown. While the energetic position of $I_{\min}^{\mathrm{DBC}}$ moves towards $E/V = -3.33$ found for PBC, the absolute value seems to saturate at $I_{\min}^{\mathrm{DBC}} = 0.592$ which is smaller than the calculated $I_{\min}^{\mathrm{PBC}} = 0.6$.

Now, we address the question, whether the critical level spacing distributions differ for different boundary conditions. In Fig. 2 the semi-logarithmic plots of $P(s)$ are shown for PBC with system size $L/a = 128$ and for DBC with $L/a = 256$. A small, but significant difference is observed between the curves for PBC and DBC for spacings $s > 2$. In contrast to the 3d orthogonal and the 2d symplectic case the large-$s$ slope is steeper for DBC than for PBC in the QHE system. Within the numerical uncertainty of our data, no size dependence could be detected for $P(s)$ with PBC in the range from $L/a = 32$ to 128 and for DBC in the range $L/a = 128$ up to 256. Therefore, we have to conclude that for the QHE system in the limit $L \to \infty$ the critical distributions are different for PBC and DBC, but the critical energies are the same. The reason for this peculiar behavior presumably originates in the different topology of the systems [42]. However, in order to detect the small difference in $P(s)$, strong finite size effects have to be overcome. The latter are due to the appearance of extended edge states that reside along the boundaries of the QHE system when DBC are applied.

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